

## Tower of Hanoi – The Temple Challenge (Stage One)

Every day a different monk will go to the Temple. He'll look at the tower and know exactly which piece to move next. No communication between the monks is required. Reasoning is their only friend. Would you like to know the secret?

The monks are familiar with the usual rules of playing the Tower of Hanoi and you must be too. Only then will you be ready to learn the five secret steps here.



### Step One – Where is the largest piece?

Have you noticed that the largest piece can only appear in two positions? It's either at START or FINISH.

This game has two stages.

In the first stage, the largest piece is at the START position and we are working to free the largest piece.

In the second stage, the largest piece is at the FINISH position and we are working to build a stack above it.

Where is the largest piece in #17 below?



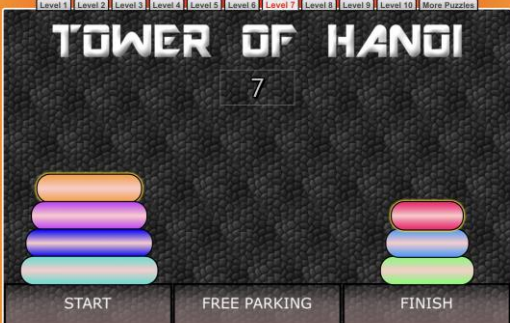





The largest piece (turquoise) is on START. We are in the first stage.

## Step Two – Are we at a milestone?

The game involves a number of milestones. The milestones for the first stage are shown below.

Note that the first stage of the game passes through the milestones in ascending order from milestone 1A to 2A to 3A etc.

First Stage Milestones	
<p><b>START</b></p>	<p><b>MILESTONE 1A</b> (Pink Milestone 1A)</p> <p>I have a Level 1 stack ↓ All other pieces are at START</p>
	
<p><b>MILESTONE 2A</b> (Pale Blue Milestone 2A)</p> <p>I have a Level 2 stack ↓ All other pieces are at START</p>	<p><b>MILESTONE 3A</b> (Green Milestone 3A)</p> <p>I have a Level 3 stack ↓ All other pieces are at START</p>
	

First Stage Milestones	
<p><b>MILESTONE 4A</b> (Orange Milestone 4A)</p> <p>I have a Level 4 stack ↓ All other pieces are at START</p>	<p><b>MILESTONE 5A</b> (Purple Milestone 5A)</p> <p>I have a Level 5 stack ↓ All other pieces are at START</p>
	
<p><b>MILESTONE 6A</b> (Dark Blue Milestone 6A)</p> <p>I have a Level 6 stack ↓ All other pieces are at START</p>	<p>H</p>
	

A monk will be very pleased if he visits the temple and finds that the tower is at any of these milestones. Can you see why?

He will simply move the available piece from START to the empty base and his work is done.

What if the tower isn't at a milestone?



## Step Three – Which milestones are we between?

Identify the two largest pieces that are **not** on START.

We'll call these the **key pieces**.



For #17, these are orange and purple

These tell us that our previous milestone was Orange Milestone 4A and that the next milestone is Purple Milestone 5A.

You can go straight to Step Four now or ponder a little longer about how we know this.

Purple only leaves START after Orange Milestone 4A has been built. Purple has left START so we must have past Orange Milestone 4A.

Dark Blue can't leave START until Purple Milestone 5A has been built. Dark Blue is still on START so we haven't past Purple Milestone 5A yet.

## Step Four – Are the key pieces on the same stack?

### Example One #25



Yes, Orange and Purple are both on FINISH

As we move between milestones, we must first deconstruct the previous milestone piece-by-piece and then build the next one.



We'll deconstruct Orange Milestone 4A and then build Purple Milestone 5A.

Step Four tells us whether we're deconstructing the previous milestone or building the next one.

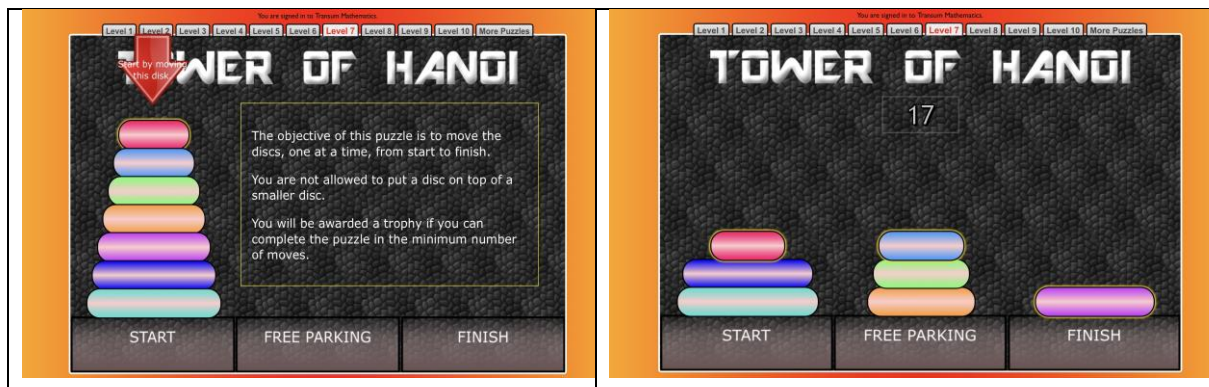
Are we deconstructing Orange Milestone 4A or building Purple Milestone 5A?

When the key pieces are on the same stack, we have commenced building the next milestone.

In Example One #25, the key pieces are on the same stack, so we've started to build Purple Milestone 5A

Ignore the small pink piece which we can also see at FINISH, it isn't relevant here.

## Example Two #17



Are the key pieces on the same stack?

No, Orange is on FREE PARKING and Purple is on FINISH.

We're still deconstructing the previous Orange Milestone 4A.

## Step Five - Journey's End

### Example One: #25



Let's recap where we're up to:

From Step Three (Identify the key pieces), we know that we're between Orange Milestone 4A and Purple Milestone 5A

From Step Four, we know that we're already building Purple Milestone 5A (same stack).

Which piece should go above orange? Green.

Green needs to be free to move and orange needs to be free to receive the green.

The green and orange pieces are currently blocked by the pieces above them. We can remove these blockages by moving the pale blue piece and the pink piece to FREE PARKING.

At that stage, the game will be at #27 (see above) and green will be ready to move to FINISH.

### **What's the next move for #25?**

Pale Blue to free parking.

### **Example Two #17**



Let's recap where we're up to:

From Step Three (Identify the key pieces), we know that we're between Orange Milestone 4A and Purple Milestone 5A

From Step Four, we know that we're deconstructing Orange Milestone 4A (different stacks).

In fact, we can see that we've only removed the small pink piece.

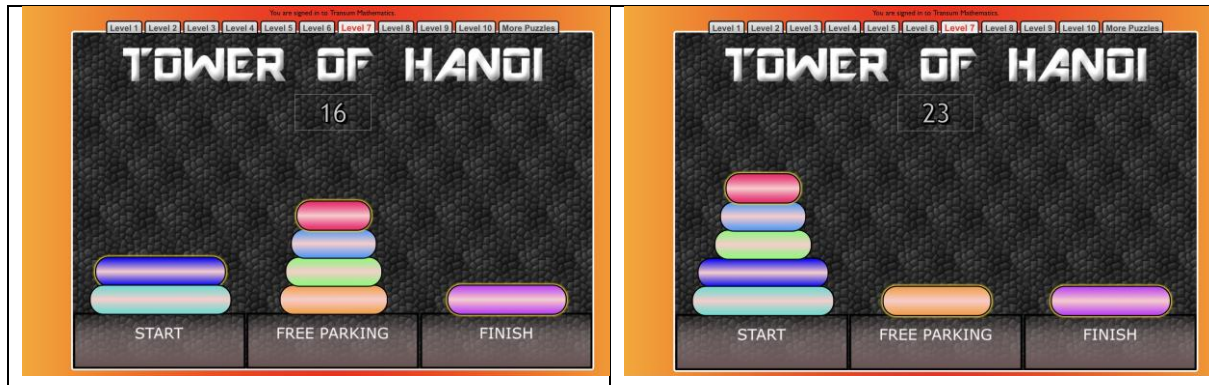
### **What's the next move?**

Pale blue will be the next piece to be removed and will go to FINISH.



## A footnote

The monk yesterday was faced with Tower Challenge #16. He did all of the hard work for us.



He knew that he had to deconstruct Milestone 4A. He just had to decide where to put the small pink piece.

He knew that eventually the orange piece would need to be placed above the purple piece on FINISH.

Therefore, orange would need to be free to move and purple would need to be free to receive the orange. At that stage the game will be at #23 (shown above).

Under the normal rules of deconstruction, we follow an alternating pattern that is determined by the green piece. Where does the green piece need to go?

Green must go to START as shown in #23 above. Therefore pale blue must go to FINISH and pink to START.

Therefore the wise monk yesterday moved the pink piece to START.

## References:

Transum – Tower of Hanoi

[https://www.transum.org/Maths/Investigation/Tower\\_Of\\_Hanoi/](https://www.transum.org/Maths/Investigation/Tower_Of_Hanoi/)

How to play the tower of Hanoi

All Math Considered – A math blog by Dan Ma

<https://allmathconsidered.wordpress.com/2016/10/>