Simultaneous Equations Extension Exercise

Are you are you as skilled as students were over 100 years ago? This exercise appeared in A First Book in Algebra by Wallace C Boyden published in 1895

Solve:

1. $\begin{cases} x + y = 4, \\ 3x - 2y = 7. \end{cases}$

2. $\begin{cases} x - y = 2, \\ 2x + 5y = 18. \end{cases}$

3. $\begin{cases} 5x + 2y = 47, \\ 2x - y = 8. \end{cases}$

4. $\begin{cases} 4x - 3y = 10, \\ 6x + 4y = 49. \end{cases}$

5. $\begin{cases} 8x - 2y = 6, \\ 10x + 7y = 36. \end{cases}$

6. $\begin{cases} 2x - 5y = -11, \\ 3x + y = 9. \end{cases}$

7. $\begin{cases} 7x - 3y = 41, \\ 2x + y = 12. \end{cases}$

8. $\begin{cases} 2x + 9y = -5, \\ 11x + 15y = 7. \end{cases}$

9. $\begin{cases} 4y - 2x = 4, \\ 10y + 3x = -8. \end{cases}$

10. $\begin{cases} 3x - 5y = 15, \\ 5x + 3y = 8. \end{cases}$

11. $\begin{cases} 3y - 2x = 3, \\ 4y - 6x = 2\frac{1}{3}. \end{cases}$

12. $\begin{cases} 3x + 2y = 11, \\ 7x - 5y = 190. \end{cases}$

13. $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 11, \\ 8x + \frac{2}{5}y = 102. \end{cases}$

14. $\begin{cases} 5x + 2y = 66, \\ \frac{x}{3} + \frac{3y}{4} = 15\frac{1}{2}. \end{cases}$

15. $\begin{cases} \frac{3x}{5} - \frac{2y}{7} = 35, \\ x + 2y = -63. \end{cases}$

16. $\begin{cases} x - \frac{3y}{5} = 6, \\ \frac{2x}{3} + 7y = 189. \end{cases}$

17. $\begin{cases} \frac{x+2y}{3x-y} = 1, \\ \frac{4y-x}{3+x-2y} = 2\frac{1}{2}. \end{cases}$

18. $\begin{cases} \frac{x+2y}{x-2} = -5\frac{2}{3}, \\ \frac{2y-4x}{3-y} = -6. \end{cases}$

19. $\begin{cases} y - \frac{2y+x}{3} = \frac{2x+y}{4} - 8\frac{3}{4}, \\ \frac{3x+y}{2} - \frac{y}{3} = \frac{109}{10} + \frac{4y-x}{5}. \end{cases}$

20. $\begin{cases} x + y = a, \\ x - y = b. \end{cases}$

21. $\begin{cases} \frac{3x-19}{2} + 4 = \frac{3y+x}{3} + \frac{5x-3}{2}, \\ \frac{4x+5y}{16} + \frac{2x+y}{2} = \frac{9x-7}{8} + \frac{3y+9}{4}. \end{cases}$

22. $\begin{cases} \frac{1}{5}(3x - 2y) + \frac{1}{3}(5x - 3y) = x, \\ \frac{4x-3y}{2} + \frac{2}{3}x - y = 1 + y. \end{cases}$

23. If 1 is added to the numerator of a fraction, its value is $\frac{1}{8}$; but if 4 is added to its denominator, its value is $\frac{1}{4}$. What is the fraction?

    Suggestion. Letting x equal the numerator, and y the denominator, form two equations.

24. If 2 is subtracted from both numerator and denominator of a certain fraction, its value is $\frac{3}{5}$; and if 1 is added to both numerator and denominator, its value is $\frac{2}{3}$. What is the fraction?

25. If 2 is added to both numerator and denominator of a certain fraction, its value is $\frac{2}{3}$; but if 3 is subtracted from both numerator and denominator, its value is $\frac{1}{2}$. What is the fraction?

26. If 3 be subtracted from the numerator of a certain fraction, and 3 be added to the denominator, its value will be $\frac{1}{2}$; but if 5 be added to the numerator, and 5 be subtracted from its denominator, its value will be 2. What is the fraction?

27. The sum of two numbers divided by 2 is 43, and their difference divided by 2 is 19. What are the numbers?

28. The sum of two numbers divided by 3 gives as a quotient 30, and their difference divided by 9 gives 4. What are the numbers?

29. Five years ago the age of a father was four times that of his son; five years hence the age of the father will be $2\frac{1}{3}$ times that of the son. What are their ages?

30. Seven years ago John was one-half as old as Henry, but five years hence he will be three-quarters as old. How old is each?

31. A and B own herds of cows. If A should sell 6 cows, and B should buy 6, they would have the same number; if B should sell 4 cows to A, he would have only half as many as A. How many cows are there in each herd?

32. The cost of 5 pounds of tea and 7 pounds of coffee is \$4.94; the cost of 3 pounds of tea and 6 pounds of coffee is \$3.54. What is the cost of the tea and coffee per pound?

33. What is the price of corn and oats when 4 bushels of corn with 6 bushels of oats cost \$4.66, and 5 bushels of corn with 9 bushels of oats cost \$6.38?

Answers

1. $x = 3,\ y = 1.$
2. $x = 4,\ y = 2.$
3. $x = 7,\ y = 6.$
4. $x = 5\frac{1}{2},\ y = 4.$
5. $x = 1\frac{1}{2},\ y = 3.$
6. $x = 2,\ y = 3.$
7. $x = 5,\ y = -2.$
8. $x = 2,\ y = -1.$
9. $x = -2\frac{1}{4},\ y = -\frac{1}{8}.$
10. $x = 2\frac{1}{2},\ y = -1\frac{1}{2}.$
11. $x = \frac{1}{2},\ y = 1\frac{1}{3}.$
12. $x = 15,\ y = -17.$
13. $x = 12,\ y = 15.$
14. $x = 6,\ y = 18.$
15. $x = 35,\ y = -49.$
16. $x = 21,\ y = 25.$
17. $x = 3,\ y = 2.$
18. $x = \frac{1}{2},\ y = 4.$
19. $x = 12,\ y = 15.$
20. $x = \frac{a+b}{2},\ y = \frac{a-b}{2}.$
21. $x = 39,\ y = -56.$
22. $x = -2,\ y = -1\frac{17}{21}.$
23. $\frac{7}{24}.$
24. $\frac{11}{17}.$
25. $\frac{8}{13}.$
26. $\frac{11}{13}.$
27. 24; 62.
28. 27; 63.
29. 13; 37.
30. J., 13 yrs.; H., 19 yrs.
31. A, 36 cows; B, 24 cows.
32. tea, 54; coffee, 32.
33. corn, 61; oats, 37.